Trigonometry Workbook Solutions

The unit circle

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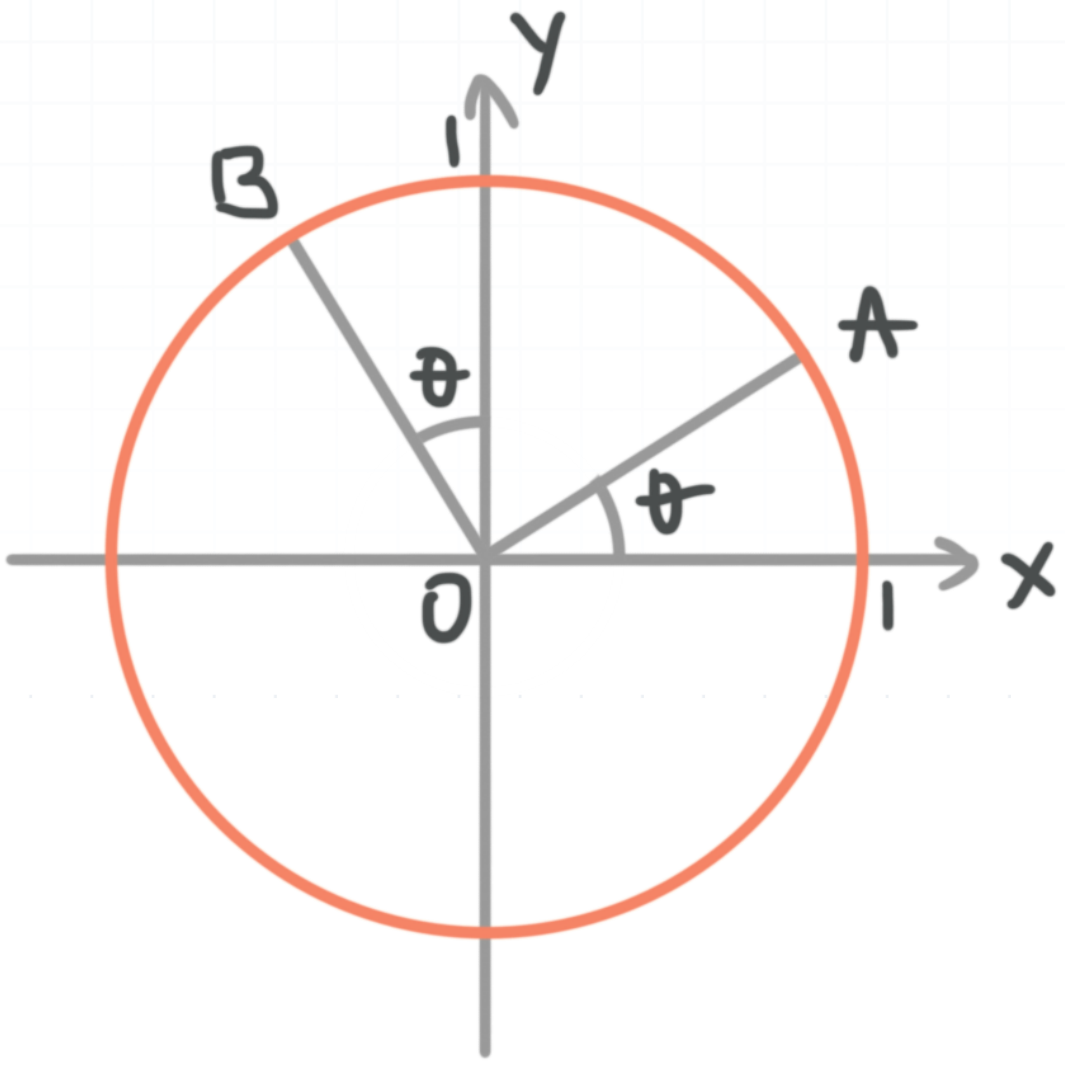
MATH

THE UNIT CIRCLE

■ 1. The points A and B lie on the unit circle in quadrants I and II respectively. The angle between OA and the positive direction of the x-axis is θ. The angle between OB and the positive direction of the y-axis is θ. In other words, these two angles have the same measure. Find the sine of angle AOB.



Solution:

The angle between OB and the positive direction of the x-axis is $90^\circ + \theta$. So the angle AOB is $(90^\circ + \theta) - \theta = 90^\circ$.

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Because we know $\sin 90^\circ = 1$,

$$\sin AOB = \sin 90^\circ = 1$$

■ 2. Find the coordinates of a point P which is the point where the terminal side of the angle $\theta = 60^\circ$ intersects the circle $x^2 + y^2 = 4$.

Solution:

First, find the coordinates of a point P_u where the terminal side of θ intersects the unit circle. Find the coordinates of P_u. In the unit circle,

the x-coordinate of P_u is $\cos\theta = \cos 60^\circ = \dfrac{1}{2}$

the y-coordinate of P_u is $\sin\theta = \sin 60^\circ = \dfrac{\sqrt{3}}{2}$

We can sketch a diagram.

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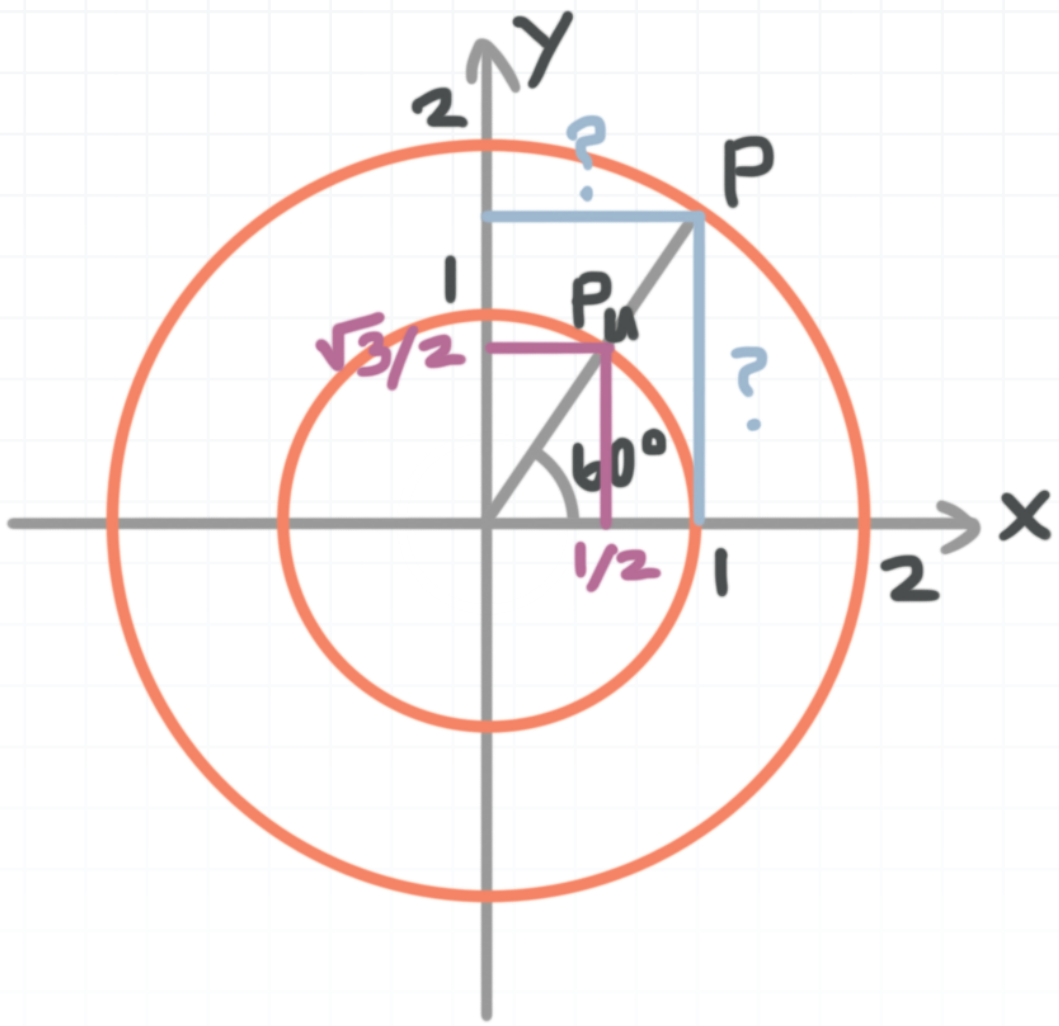

Since the radius of the circle $x^2 + y^2 = 4$ is 2, we can just multiply the coordinates of P_u by 2 to get coordinates of P.

$$2P_u = 2\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) = \left(\frac{2}{2}, \frac{2\sqrt{3}}{2}\right) = (1, \sqrt{3})$$

■ 3. For some angle $\theta \in [0, \pi/2)$, its sine and cosine have a ratio of $3 : 4$. Find the coordinates of a point P which is located where the terminal side of θ intersects the unit circle.

Solution:

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Let (x, y) be the coordinates of P. By the definition, $\sin\theta$ is the y-coordinate of P and $\cos\theta$ is the x-coordinate of P. Since sine and cosine are in a ratio of $3:4$, y and x are in a ratio of $3:4$, which means that there exists a real number k where

$$y = 3k$$

$$x = 4k$$

Substitute these values of x and y into the equation of the unit circle.

$$x^2 + y^2 = 1$$

$$(4k)^2 + (3k)^2 = 1$$

$$25k^2 = 1$$

$$k^2 = \frac{1}{25}$$

Since θ lies in quadrant I, x and y are positive numbers, and so k is also positive

$$k = \sqrt{\frac{1}{25}} = \frac{1}{5}$$

Then the values of x and y are

$$x = 4k = \frac{4}{5}$$

$$y = 3k = \frac{3}{5}$$

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FINDING SINE AND COSINE OF AN ANGLE THETA

■ 1. Use the unit circle to find all possible values of θ.

$$|\cos\theta| = \cos 39^\circ$$

Solution:

The absolute value of $\cos\theta$ means we need to consider two cases:

1. $\cos\theta = \cos 39^\circ$

2. $\cos\theta = -\cos 39^\circ$

The value $\cos 39^\circ$ is equal to the x-value where $\cos 39^\circ$ intersects the unit circle. But because $\cos 39^\circ$ just gives an x-value, we'll also find that same x-value in quadrant IV when we reflect $\cos 39^\circ$ over the x-axis.

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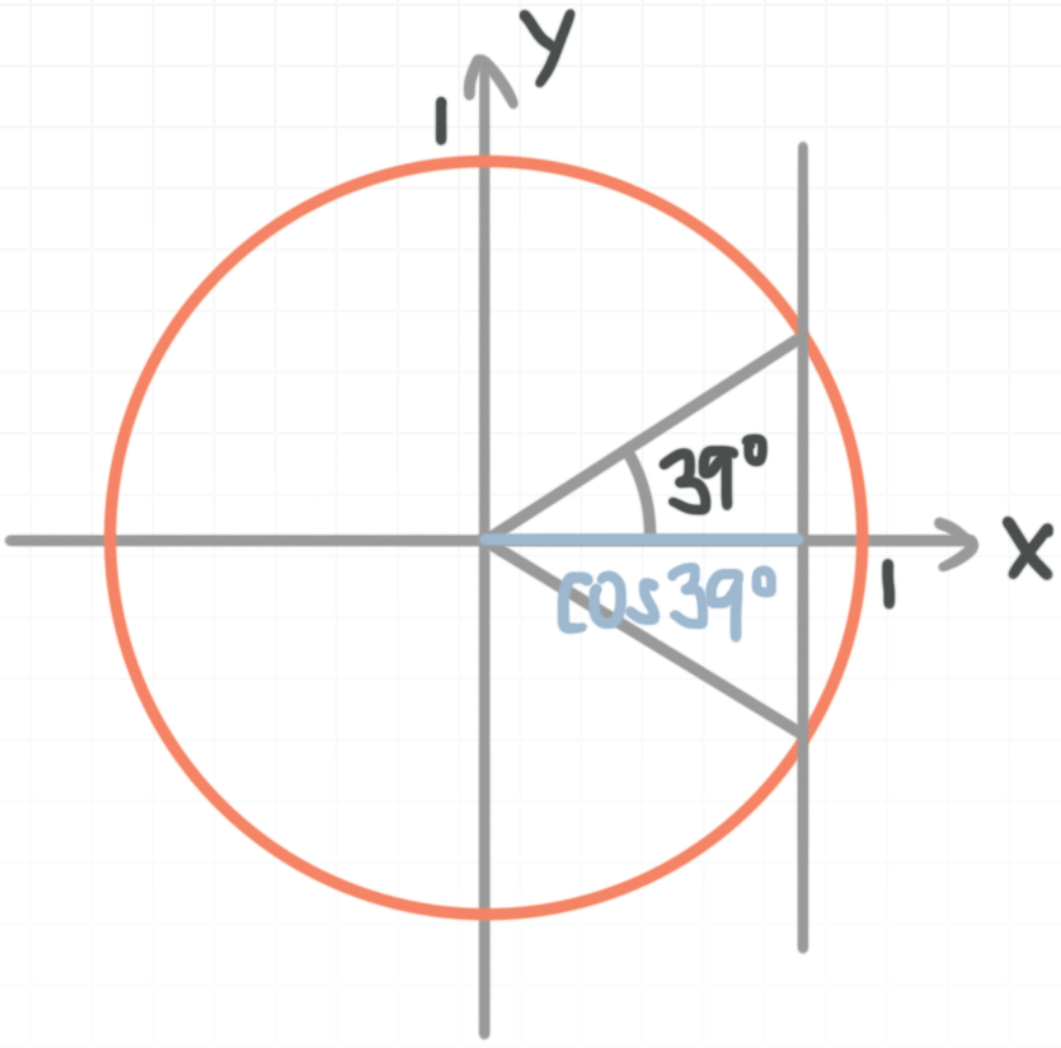

The measure of that angle will be

$$360° - 39° = 321°$$

So the angles θ where $\cos\theta = \cos 39°$ are $39°$ and $321°$.

The value $-\cos 39°$ is the angle we get when we reflect $\cos 39°$ over the y-axis into quadrant II. But because $-\cos 39°$ just gives an x-value, we'll also find that same x-value in quadrant III when we reflect $-\cos 39°$ from quadrant II over the x-axis into quadrant III.

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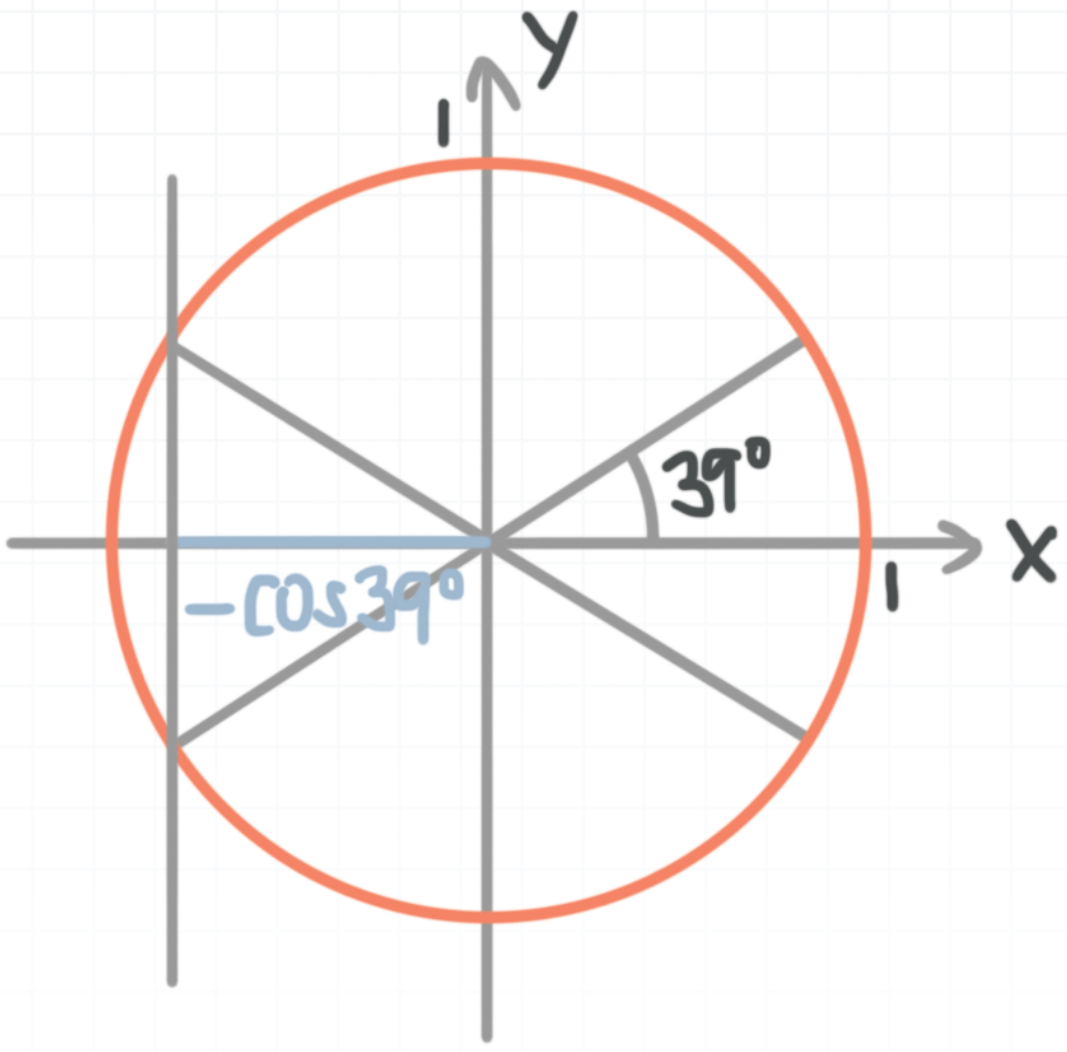

These angles will be

II quadrant: $180^\circ - 39^\circ = 141^\circ$

III quadrant: $180^\circ + 39^\circ = 219^\circ$

So the angles θ such that $\cos\theta = -\cos 39^\circ$, are 141° and 219°.

■ 2. The terminal side of the angle θ intersects the unit circle at $P(-0.8,0.6)$. Find the values of all six circular functions of θ.

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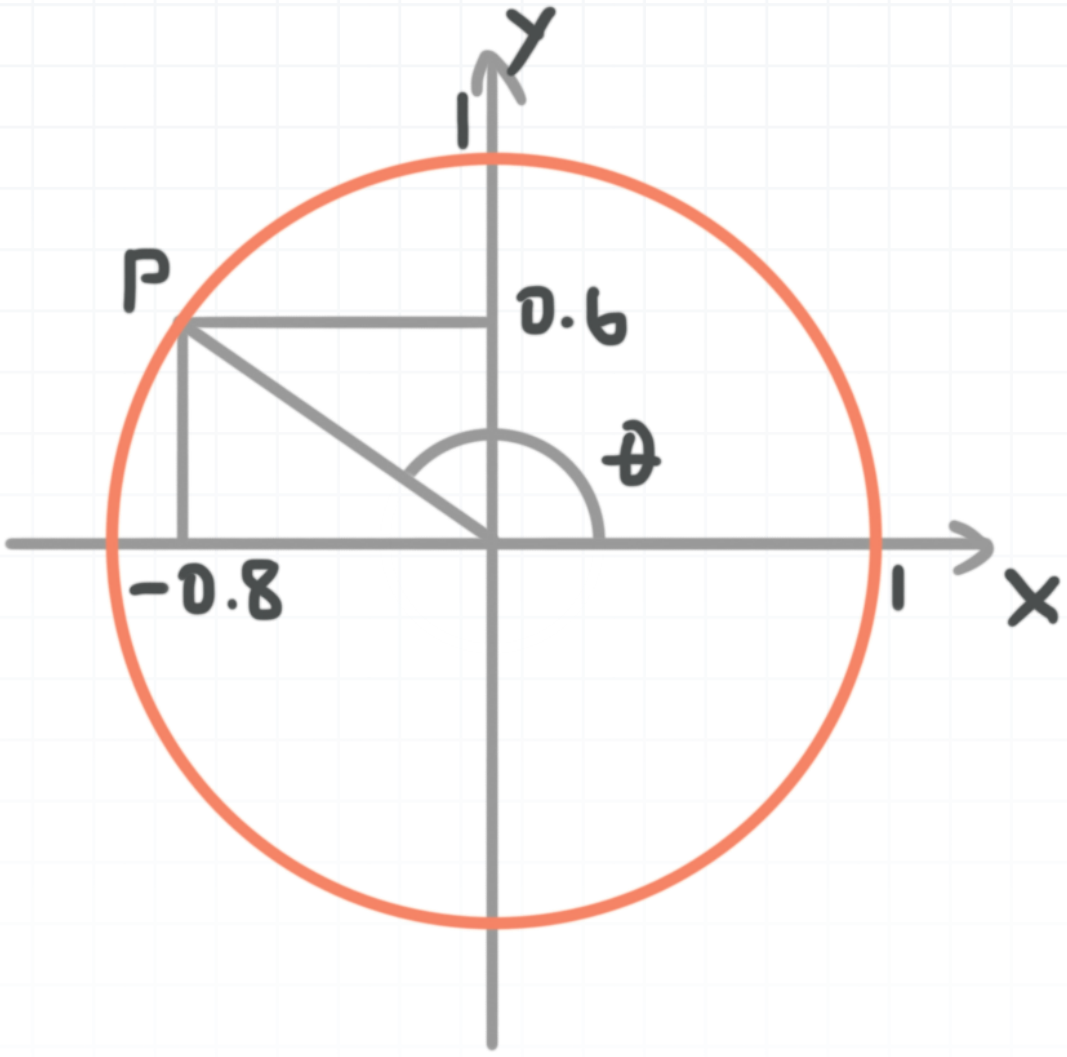

Solution:

Although the problem may look hard, it can be easily solved using the fact that for the angle θ, the value of $\cos\theta$ is equal to the x-coordinate of the point P, and the value of $\sin\theta$ is equal to its y-coordinate.

$$\sin\theta = 0.6$$

$$\cos\theta = -0.8$$

Then find the values of the other circular functions using their definitions in terms of sine and cosine.

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$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{0.6}{-0.8} = -\frac{3}{4}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{-0.8}{0.6} = -\frac{4}{3}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-0.8} = -\frac{5}{4}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{0.6} = \frac{5}{3}$$

■ 3. Consider the angles $10°$, $100°$, $190°$, and $280°$. Sort them in ascending order of their cosines.

Solution:

Sketch out these angles in the unit circle.

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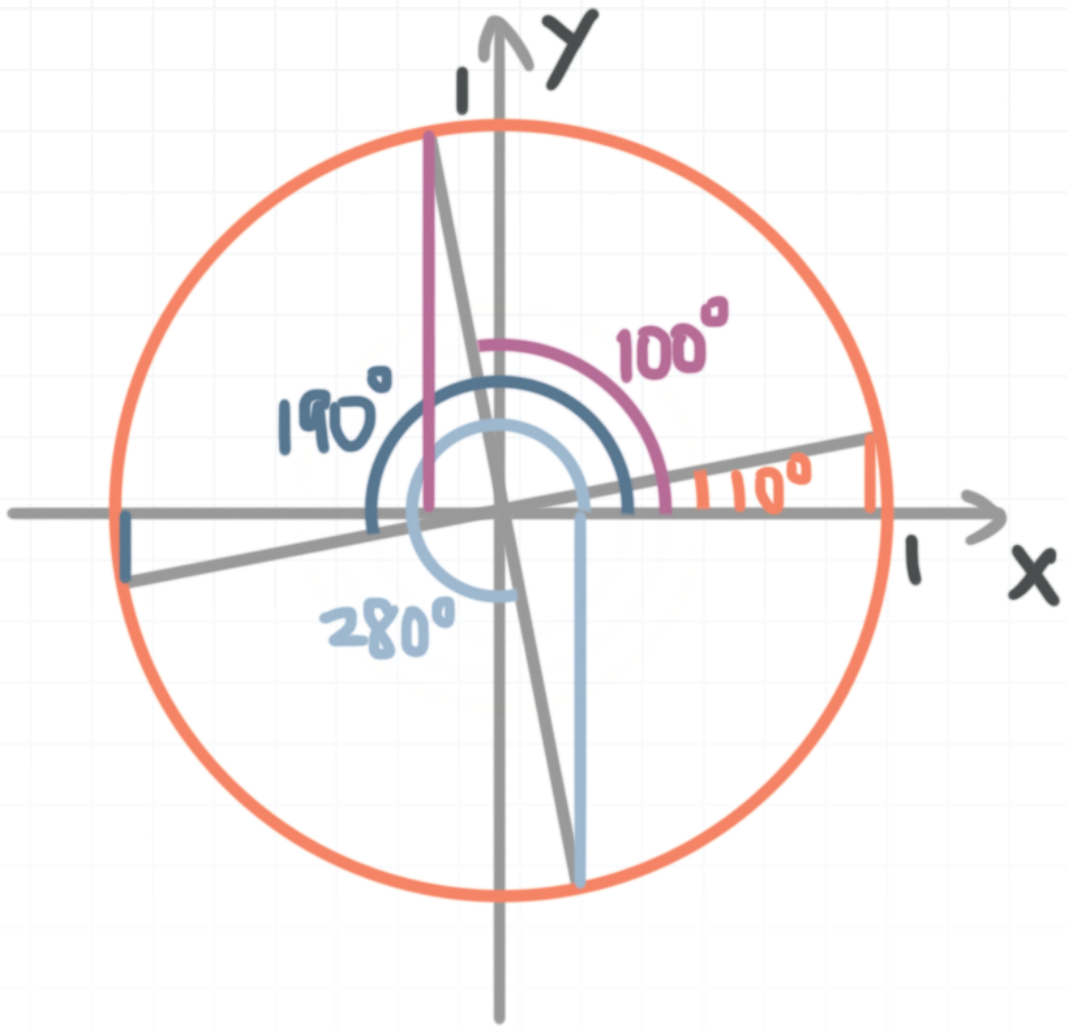

We know that the value of $\cos\theta$ is equal to the x-coordinate of the point at which the terminal side of θ intersects the unit circle. From the sketch, we can see that

$\cos 10^\circ$ is a positive number close to 1,

$\cos 100^\circ$ is a negative number close to 0,

$\cos 190^\circ$ is a negative number close to -1, and

$\cos 280^\circ$ is a positive number close to 0.

So

$$\cos 190^\circ < \cos 100^\circ < \cos 280^\circ < \cos 10^\circ$$

and the angles should be sorted as

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$190°$, $100°$, $280°$, and $10°$

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FINDING SINE GIVEN COSINE, OR VICE VERSA

■ 1. Find $\sin\theta$ if $\theta \in [0,180°]$. Hint: make a substitution into the Pythagorean identity $\sin^2\theta = 1 - \cos^2$.

$$\cos^2\theta - 0.36 = 0$$

Solution:

Rewrite the equation.

$$\cos^2\theta - 0.36 = 0$$

$$\cos^2\theta = 0.36$$

Substitute into the Pythagorean identity $\sin^2\theta = 1 - \cos^2$.

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - 0.36$$

$$\sin^2\theta = 0.64$$

Since $\sin\theta \geq 0$ for any $\theta \in [0,180°]$,

$$\sin\theta = \sqrt{0.64}$$

$$\sin\theta = 0.8$$

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■ 2. Find $\sin\theta$ and $\sin\phi$ if $\theta, \phi \in [0,2\pi)$.

$$\cos\theta + \cos\phi = 2$$

Solution:

Since the range of the cosine function is $[-1,1]$, the sum of two cosines is equal to 2 only if each of them is equal to 1. So

$$\cos\theta = 1$$

$$\cos\phi = 1$$

The equation for θ only one solution on $[0,2\pi)$, which is $\theta = 0$. So $\sin\theta = \sin 0 = 0$.

The equation for ϕ is the same, so its solution is $\phi = 0$, and therefore $\sin\phi = 0$ as well.

■ 3. Find $\sin\theta$ if $\theta \in [0,2\pi)$. Hint: make a substitution into the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$.

$$\cos\theta = \sin\theta$$

Solution:

Start with the Pythagorean identity, then substitute.

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$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \sin^2\theta = 1$$

$$2\sin^2\theta = 1$$

$$\sin^2\theta = \frac{1}{2}$$

$$\sin\theta = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

So there are two values of sine that satisfy the given conditions.

$$\sin\theta = -\frac{\sqrt{2}}{2}$$

$$\sin\theta = \frac{\sqrt{2}}{2}$$

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ALL POSSIBLE VALUES FOR THE ANGLE THETA

■ 1. Consider the equation $f(x) = 0$, where $f(x)$ is some trigonometric function with period 2π. The equation $f(x) = 0$ has two solutions in $[0,2\pi)$, namely $\pi/4$ and $5\pi/4$. Extend these solutions to all of the periods and write them down using set notation in the simplest way possible.

Solution:

Let's use a standard way for extending the solutions to all periods by adding the term $2\pi n$:

$$x_1 = \frac{\pi}{4} + 2\pi n \text{ where } n \text{ is any integer}$$

$$x_2 = \frac{5\pi}{4} + 2\pi n \text{ where } n \text{ is any integer}$$

If we expand these solutions into a list of angles, we get

$$x_1 = \ldots, \frac{\pi}{4}, \frac{\pi}{4} + 2\pi, \frac{\pi}{4} + 4\pi, \ldots$$

$$x_2 = \ldots, \frac{\pi}{4} + \pi, \frac{\pi}{4} + 3\pi, \frac{\pi}{4} + 5\pi, \ldots$$

Combine the lists into one sorted list.

$$x = \ldots, \frac{\pi}{4}, \frac{\pi}{4} + \pi, \frac{\pi}{4} + 2\pi, \frac{\pi}{4} + 3\pi, \frac{\pi}{4} + 4\pi, \frac{\pi}{4} + 5\pi, \ldots$$

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From the combined list, we see that we could express the pattern as

$$x = \frac{\pi}{4} + \pi n, \text{ where } n \text{ is any integer}$$

In set notation, the angles that satisfy these conditions are

$$\left\{ \frac{\pi}{4} + \pi n : \ n \in \mathbb{Z} \right\}$$

2. Solve the trigonometric equation.

$$\cos(3x + 5\pi) = 0$$

Solution:

Make the substitution $\phi = 3x + 5\pi$, and rewrite the equation in terms of ϕ.

$$\cos \phi = 0$$

In order to find values of ϕ that satisfy this equation, we'll use the unit circle to find the solutions within $[0,2\pi)$, and then extend these solutions to all periods.

We know $\cos \phi$ will be 0 when the x-coordinate of a point on the unit circle is 0. And that occurs at $\phi = \pi/2$ and $\pi = 3\pi/2$.

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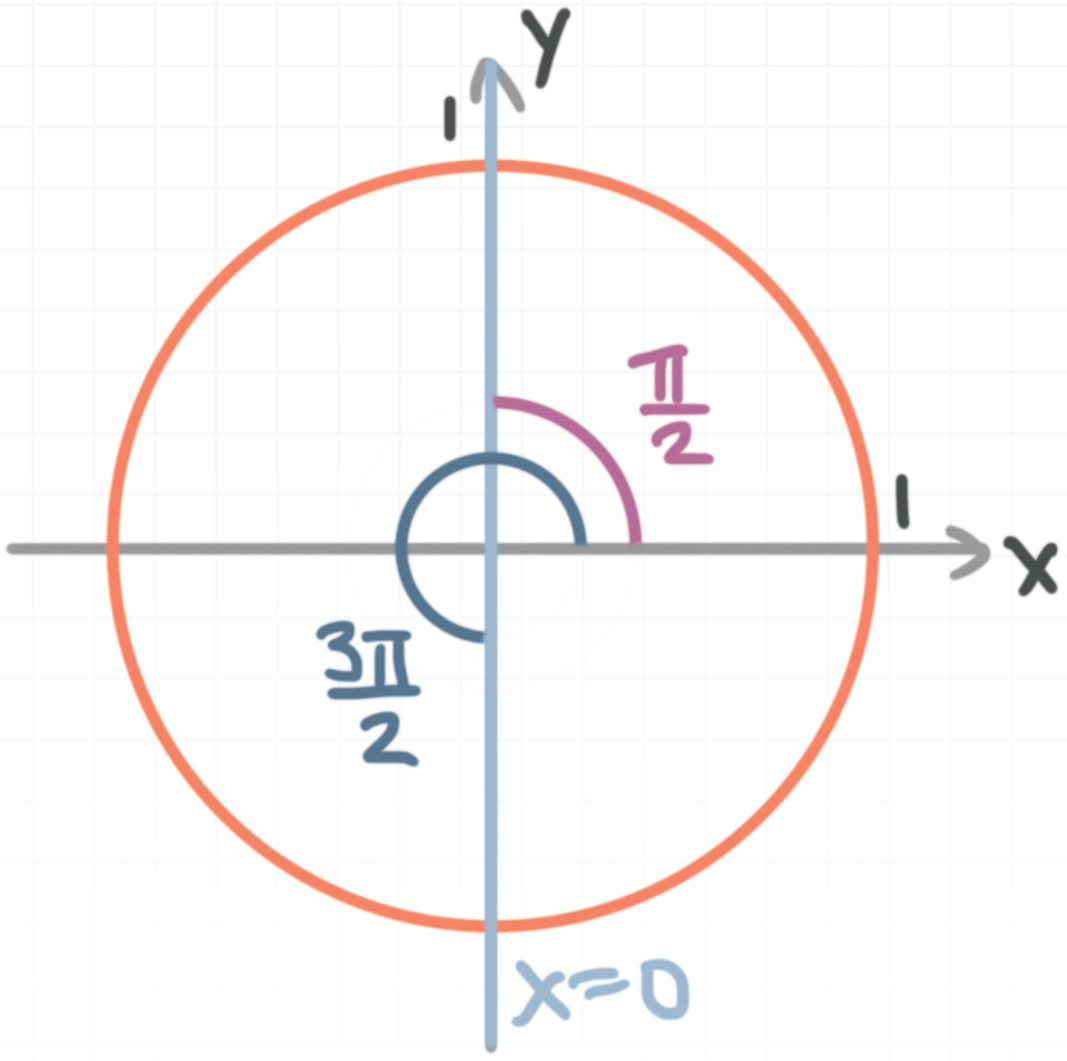

Extend these solutions to all periods by adding the term $2\pi n$.

$$x_1 = \frac{\pi}{2} + 2\pi n, \text{ where } n \text{ is any integer}$$

$$x_2 = \frac{3\pi}{2} + 2\pi n, \text{ where } n \text{ is any integer}$$

If we expand these solutions into a list of angles, we get

$$\phi_1 = \ldots, \frac{\pi}{2}, \frac{\pi}{2} + 2\pi, \frac{\pi}{2} + 4\pi, \ldots$$

$$\phi_2 = \ldots, \frac{\pi}{2} + \pi, \frac{\pi}{2} + 3\pi, \frac{\pi}{2} + 5\pi, \ldots$$

Combine the lists into one sorted list.

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$$\phi = \ldots, \frac{\pi}{2}, \frac{\pi}{2} + \pi, \frac{\pi}{2} + 2\pi, \frac{\pi}{2} + 3\pi, \frac{\pi}{2} + 4\pi, \frac{\pi}{2} + 5\pi, \ldots$$

From the combined list, we see that we could express the pattern as

$$\phi = \frac{\pi}{2} + \pi n, \text{ where } n \text{ is any integer}$$

Back substitute $\phi = 3x + 5\pi$ to put the solution set back in terms of x.

$$3x + 5\pi = \frac{\pi}{2} + \pi n$$

$$3x = \frac{\pi}{2} - 5\pi + \pi n$$

$$3x = \frac{\pi}{2} + \pi(n - 5)$$

Note that since n is any integer, $n - 5$ is also any integer. So any additional multiple of π is "consumed" by πn. Therefore, we can simplify the solution to

$$3x = \frac{\pi}{2} + \pi n$$

$$x = \frac{\pi}{6} + \frac{\pi n}{3}$$

In set notation, the angles that satisfy these conditions are

$$\left\{ \frac{\pi}{6} + \frac{\pi n}{3} : \ n \in \mathbb{Z} \right\}$$

■ 3. Find the set of points on the xy-plane that satisfy the equation.

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$$\sin x \cdot \cos y = 0$$

Solution:

A product is equal to 0 if one or both of the factors is equal to 0. So instead of one equation in terms of x and y, we can consider two simple equations, each of them in terms of one variable.

$$\sin x = 0$$

$$\cos y = 0$$

Consider the first equation. From the unit circle we know the angles where the sine function is 0 are

$$\ldots -\pi,\ 0,\ \pi,\ 2\pi,\ 3\pi, \ldots$$

$$\{\pi n :\ \ n \in \mathbb{Z}\}$$

Consider the second equation. From the unit circle we know the angles where the cosine function is 0 are

$$\ldots -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$$

$$\left\{\frac{\pi}{2} + \pi m :\ \ m \in \mathbb{Z}\right\}$$

Now we can combine the solution sets.

$$\left\{(x, y) \mid x = \ \pi n \text{ or } y = \frac{\pi}{2} + \pi m :\ \ n, m \in \mathbb{Z}\right\}$$

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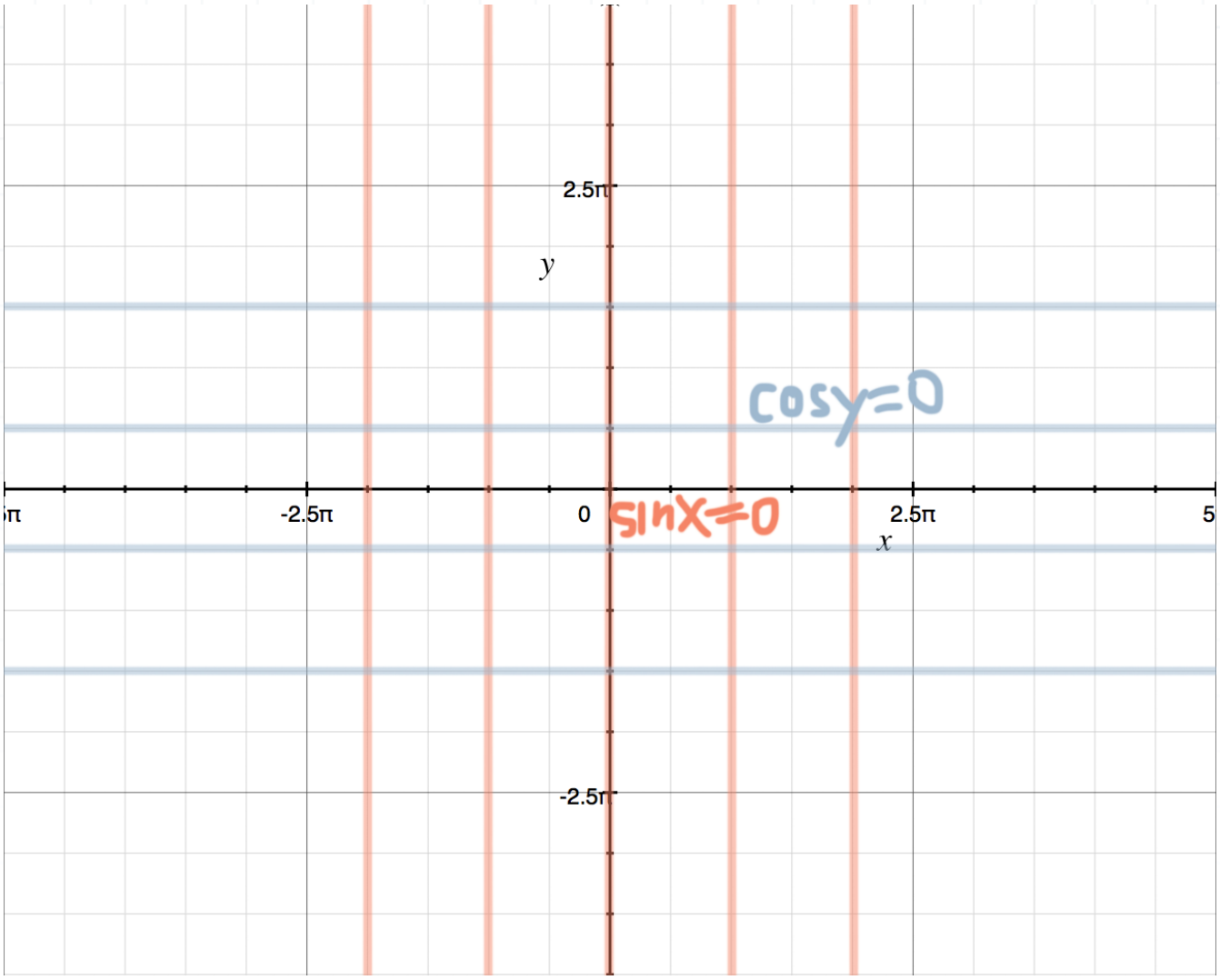
We can sketch this pattern as a graph. Looking at just the solutions that are clustered around the origin, the points that satisfy the equation will be the points where any red line intersects any blue line.



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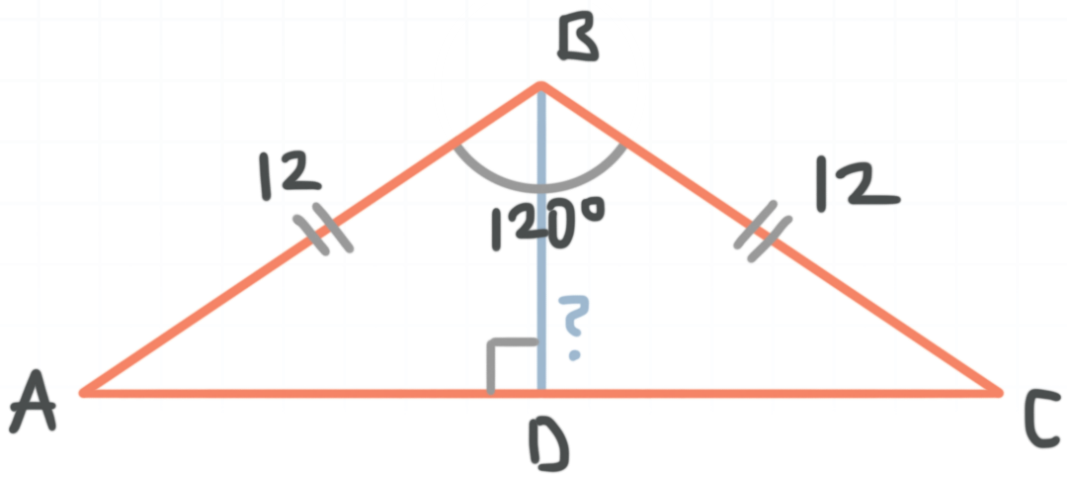
COMPLETING A RIGHT TRIANGLE

■ 1. Find the height of the isosceles triangle ABC if $AB = BC = 12$ and $m\angle B = 120°$.

Solution:

Let BD be the height of triangle ABC.



Since in an isosceles triangle the height divides an angle between legs into two equal parts, the angles ABD and CBD are both

$$\frac{120°}{2} = 60°$$

Since the height of the triangle is orthogonal to the base, the triangle ABD is a right triangle. In this triangle we know the hypotenuse $AB = 12$, the acute angle $m\angle ABD = 60°$, and we need to find the adjacent leg BD.

By the definition of cosine of an acute angle in a right triangle, it's equal to the ratio of the adjacent leg to the hypotenuse.

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$$\cos ABD = \frac{BD}{AB}$$

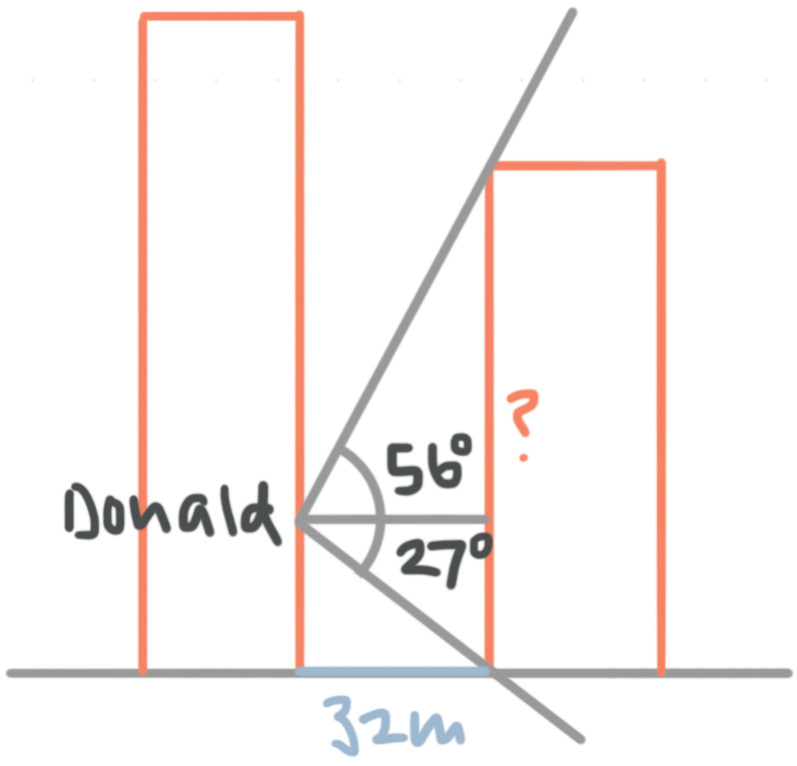
$$BD = AB \cdot \cos ABD$$

$$BD = 12 \cos 60°$$

$$BD = 12 \cdot \frac{1}{2}$$

$$BD = 6$$

2. Donald looks from his window at a neighbor's house. Donald measures the elevation angle of the highest point of the house, $52°$, and the depression angle of the lowest point of the house, $27°$. Donald knows that the distance between the houses is 32 m. Find the height of the neighbor's house.



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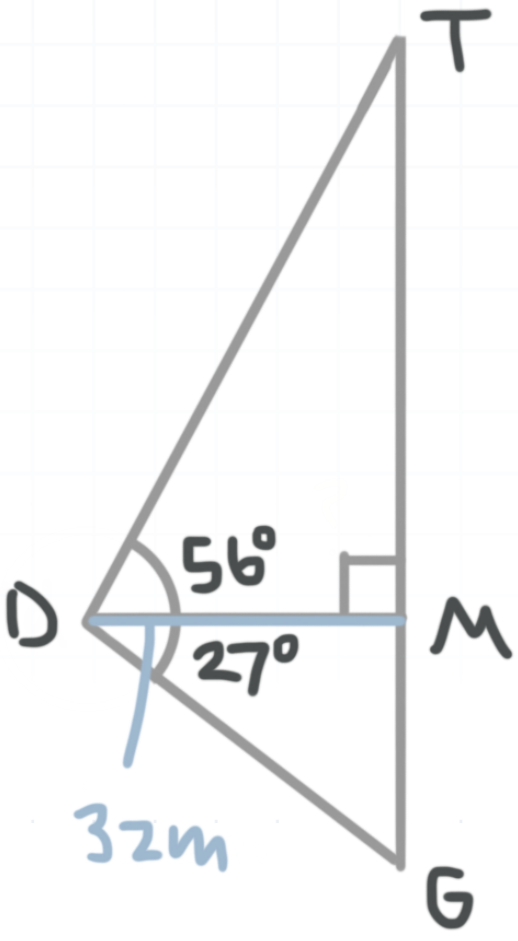
Solution:

Consider the triangle DTG, where D is Donald's position, T is the top of the house, and G is the ground. Let M be the point on the side TG such that DM is orthogonal to TG. Then DM is the distance between the houses, so it's equal to 32 m. The angle TDM is the elevation angle of $52°$, and GDM is the depression angle of $27°$.



Consider the right triangle TDM first. In the triangle, we know the acute angle $TDM = 52°$, the adjacent leg $DM = 32$ m, and we need to find the opposite leg TM.

By the definition of tangent of an acute angle in a right triangle, it's equal to the ratio of the opposite leg to the adjacent leg. Hence,

$$\tan TDM = \frac{TM}{DM}$$

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$$TM = DM \cdot \tan TDM$$

$$TM = 32 \tan 52^\circ \approx 41 \text{ m}$$

Next consider the right triangle GDM. We know the acute angle $GDM = 27^\circ$, the adjacent leg $DM = 321$, and we need to find the opposite leg GM. So we get

$$GM = DM \cdot \tan GDM$$

$$GM = 32 \tan 27^\circ \approx 16.3 \text{ m}$$

Then the height of the neighbor's house is the sum of these two values.

$$TG = TM + GM$$

$$TG = 41 + 16.3$$

$$TG \approx 57 \text{ m}$$

■ 3. A plot of land shaped like a right triangle has its hypotenuse along a river. The angle between one of the legs and the river is 30°, and the distance from the river to the furthest point of the plot is 120 m.

The land owner decides to build a fence around the plot and needs to find the total length of the fence. Since the fence is sold by sections of 10 meters, round up to the nearest 10 meters.

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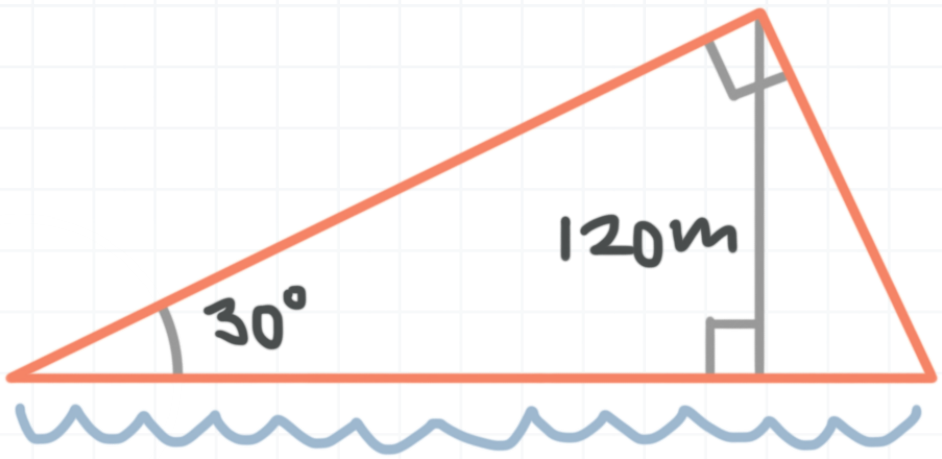
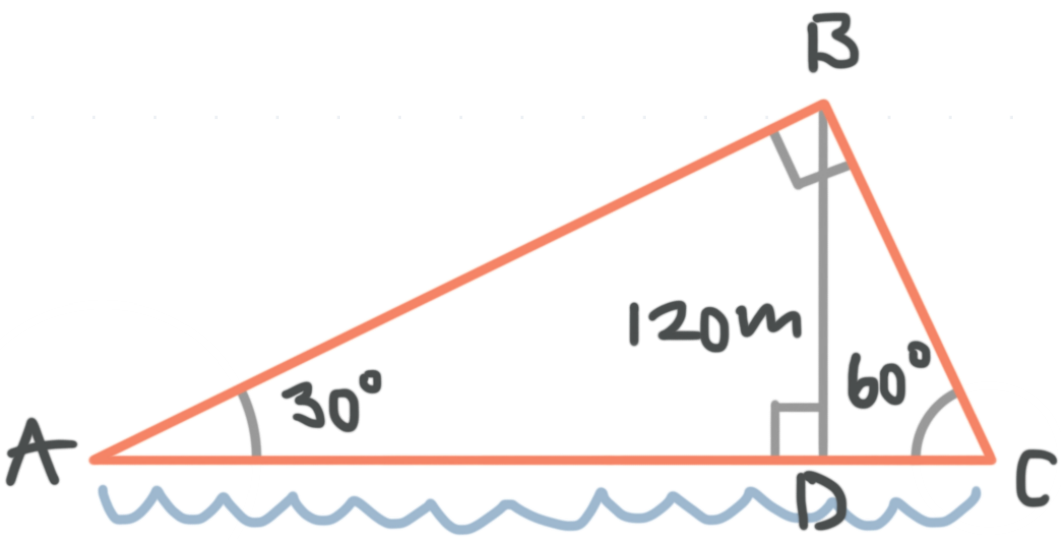


Solution:

Call the right triangle ABC, where AC is the hypotenuse along the river, and B is the right angle. Then $m\angle BAC = 30°$. And since the sum of two acute angles in a right triangle is $90°$, we can say $m\angle BCA = 90° - 30° = 60°$.

Let D be the point on side AC where BD is orthogonal to AC. Then BD is the distance from B to AC, so it's equal to 120 m.



Solve the problem in four steps:

(1) Find AB from the triangle ABD,

(2) Find BC from the triangle BCD ,

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(3) Find AC using the Pythagorean Theorem, and

(4) Find the perimeter as the sum of all sides from the previous steps.

In triangle ABD, we know the acute angle $BAD = 30°$, the opposite leg is BD, and we need to find the hypotenuse AB.

$$\sin BAD = \frac{BD}{AB}$$

$$AB = \frac{BD}{\sin BAD}$$

$$AB = \frac{120}{\sin 30°} = \frac{120}{0.5} = 240 \text{ m}$$

In triangle CBD, we know the acute angle $BCD = 60°$, the opposite leg is BD, and we need to find the hypotenuse BC.

$$BC = \frac{BD}{\sin BCD}$$

$$BC = \frac{120}{\sin 60°} \approx 139 \text{ m}$$

Use the Pythagorean theorem to find the hypotenuse AC.

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 240^2 + 139^2$$

$$AC = \sqrt{240^2 + 139^2} \approx 277 \text{ m}$$

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Now since we know the lengths of all sides of the land plot, we can find the total length of the fence by adding them together.

$$240 + 139 + 277 = 656 \text{ m}$$

Rounding up to the nearest 10 meters, the owner needs to buy 660 m of fence.

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REFERENCE ANGLE THEOREM

■ 1. Sketch the graph of $R(x)$ for $x \in [0,2\pi]$, where the value of $R(x)$ is equal to the measure of its reference angle.

Solution:

Start with the interval $[0,\pi/2)$, which is just the first quadrant. In quadrant I, the reference angle is equal to the angle itself.

$$R(x) = x$$

This is a linear function, so to sketch the graph we need to find any two points on the line. Let's take the endpoints of the interval.

$$R(0) = 0$$

$$R\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$$

So sketch a line from $(0,0)$ to $(\pi/2,\pi/2)$.

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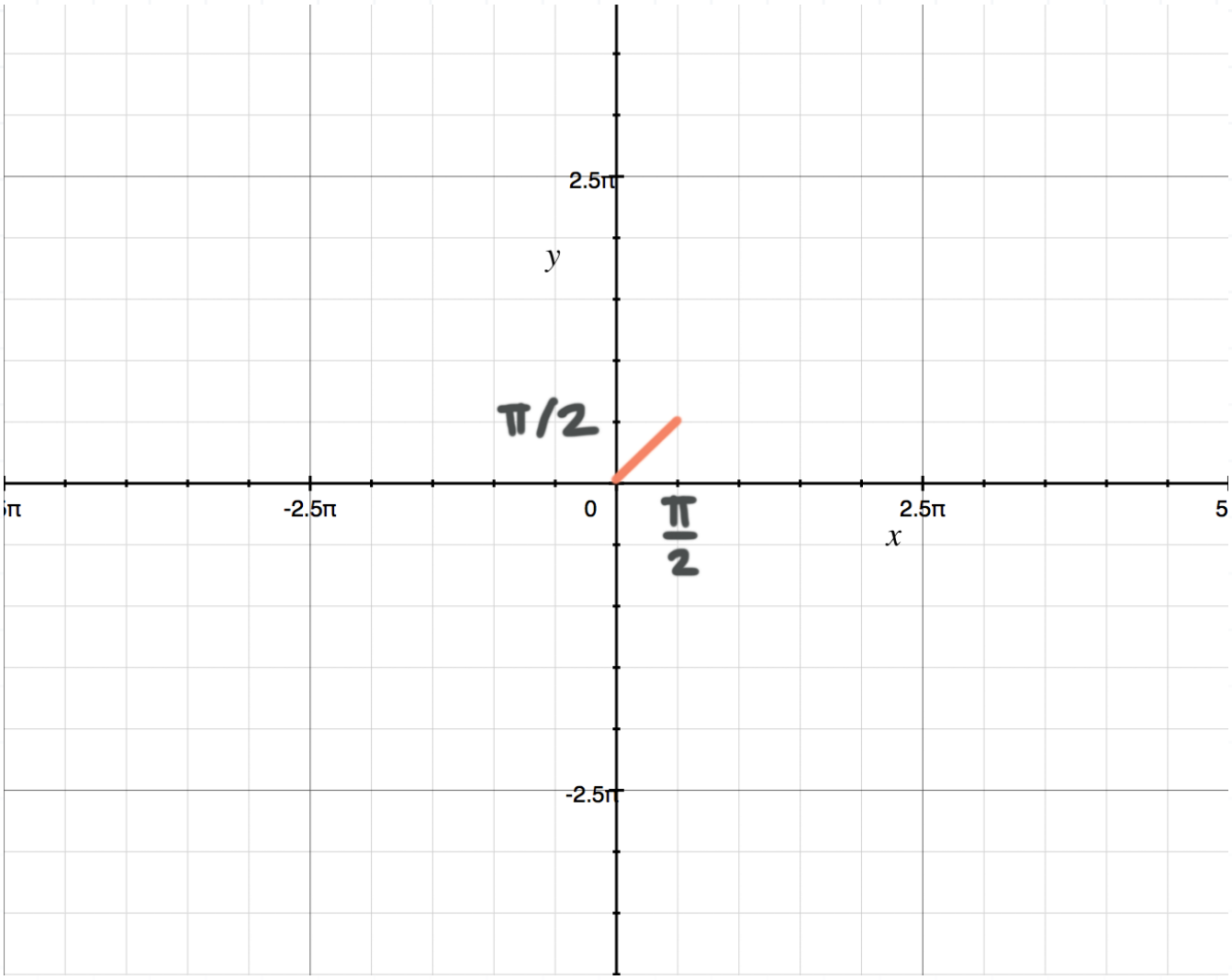

Now look at $[\pi/2,\pi)$, which is the second quadrant. In quadrant II, the reference angle of θ is equal to $\pi - \theta$.

$$R(x) = \pi - x$$

This is a linear function, so to sketch the graph we need to find any two points on the line. Let's take the endpoints of the interval.

$$R\left(\frac{\pi}{2}\right) = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

$$R(\pi) = \pi - \pi = 0$$

Sketch a line from $(\pi/2,\pi/2)$ to $(\pi,0)$.

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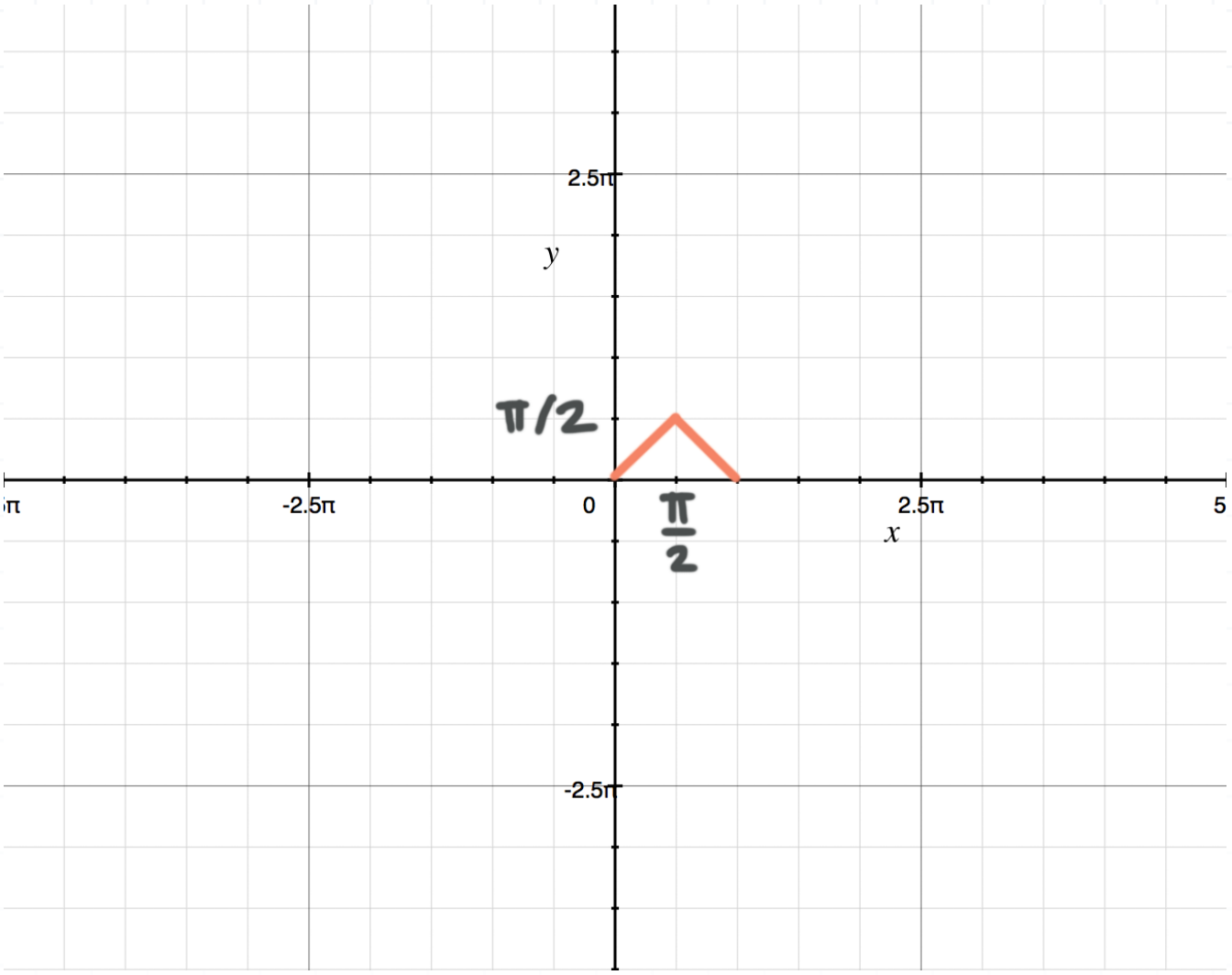

Now look at $[\pi,3\pi/2)$, which is the third quadrant. In quadrant III, the reference angle of θ is equal to $\theta - \pi$.

$$R(x) = x - \pi$$

This is a linear function, so to sketch the graph we need to find any two points on the line. Let's take the endpoints of the interval.

$$R(\pi) = \pi - \pi = 0$$

$$R\left(\frac{3\pi}{2}\right) = \frac{3\pi}{2} - \pi = \frac{\pi}{2}$$

Sketch a line from $(\pi,0)$ to $(3\pi/2,\pi/2)$.

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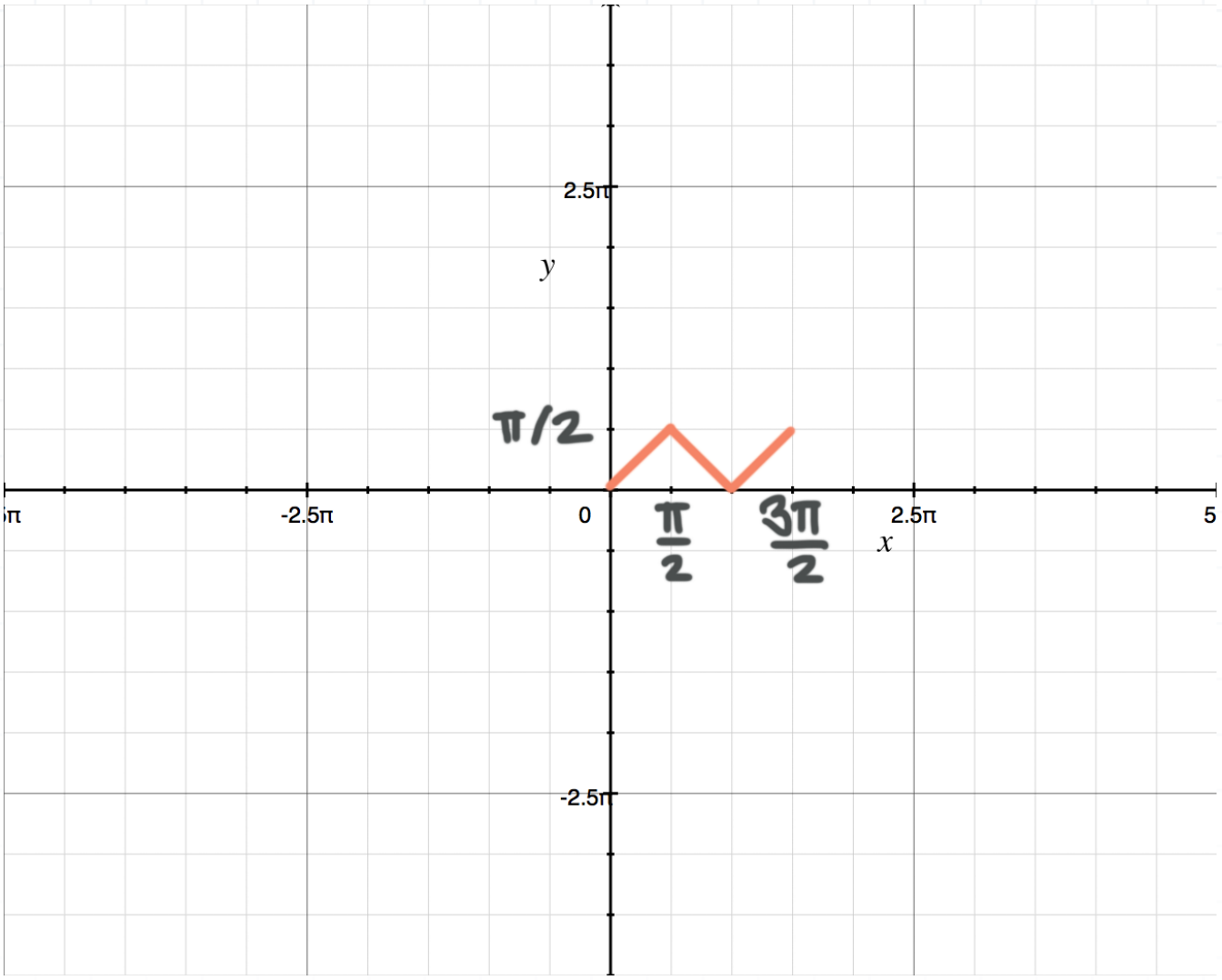

Now look at $[3\pi/2,2\pi]$, which is the fourth quadrant. In quadrant IV, the reference angle of θ is equal to $2\pi - \theta$.

$$R(x) = 2\pi - x$$

This is a linear function, so to sketch the graph we need to find any two points on the line. Let's take the endpoints of the interval.

$$R\left(\frac{3\pi}{2}\right) = 2\pi - \frac{3\pi}{2} = \frac{\pi}{2}$$

$$R(2\pi) = 2\pi - 2\pi = 0$$

Sketch a line from $(3\pi/2,\pi/2)$ to $(2\pi,0)$.

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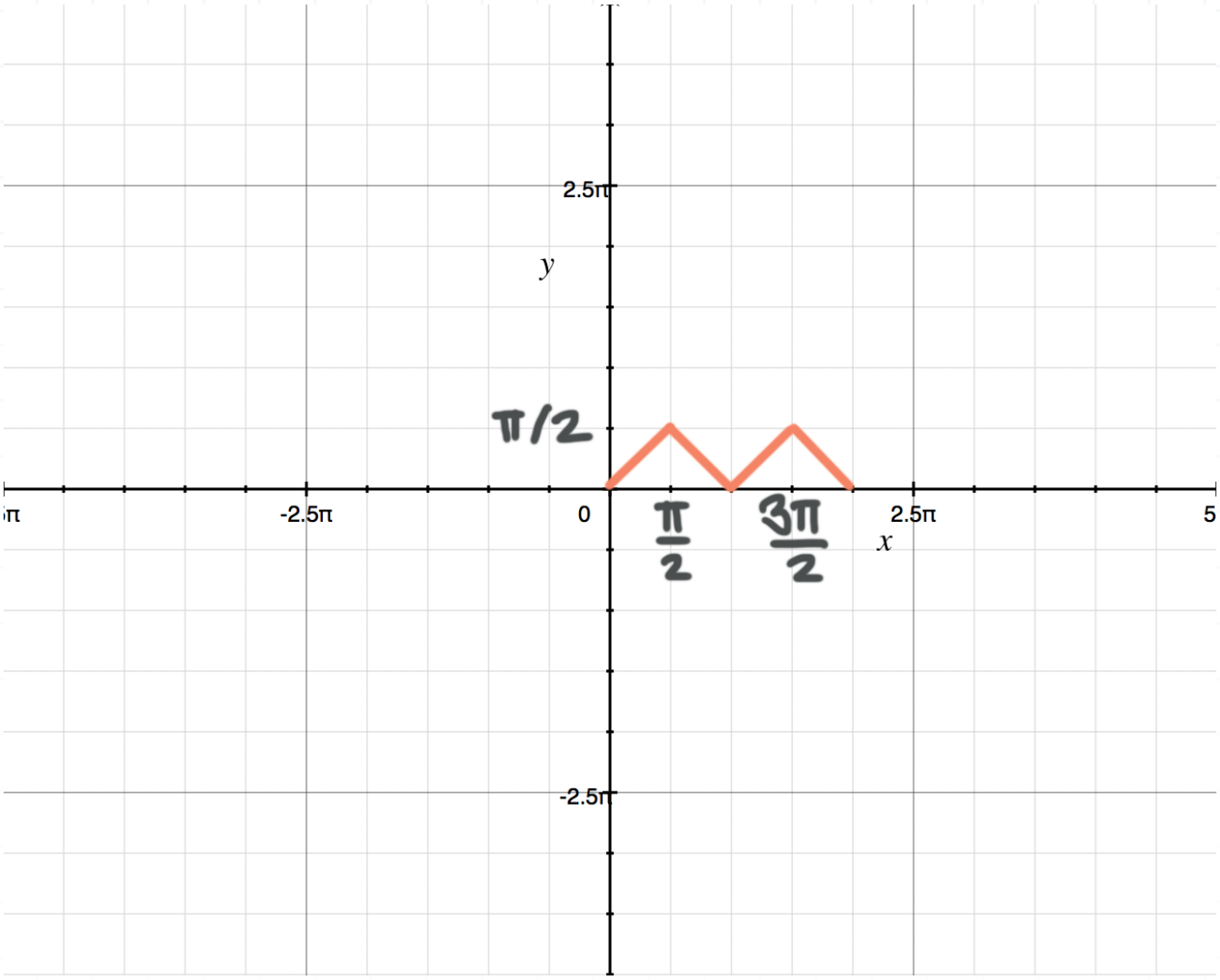

2. A trapezoid-shaped plot of land lies between the two parallel railways. The land owner needs to know the sines of all four angles of the plot, and he already knows that the sines of two angles along the same railway are 0.27 and 0.46. Find the sines of the two remaining angles.

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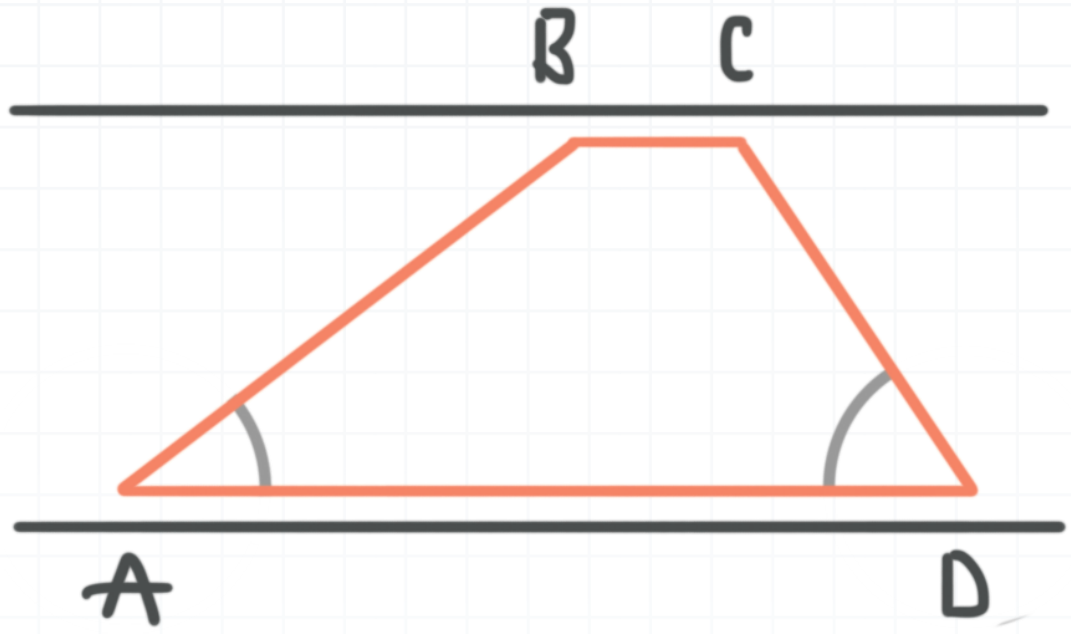

Solution:

Call the trapezoid $ABCD$ where BC and AD are the parallel sides, $\sin A = 0.27$, and $\sin D = 0.46$.

From the properties of trapezoids, we know the angles A and B are supplementary, which means they sum to $180°$.

$$B = 180° - A$$

$$C = 180° - D$$

So if we know $\sin A = 0.27$, then we need to find $\sin B = \sin(180° - A)$. Angle A lies in quadrant I, so its reference angle is A. The angle $180° - A$ lies in quadrant II, so its reference angle is $180° - (180° - A) = A$.

Therefore, since the angles A and $180° - A$ have equal reference angles, by the reference angle theorem the absolute values of their trig functions are equal (other than the fact that they might have different signs).

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Since $180° - A$ lies in quadrant II, its sine is positive and its absolute value is $\sin A = 0.27$, so

$$\sin B = \sin(180° - A)$$

$$\sin B = \sin A$$

$$\sin B = 0.27$$

If we know $\sin D = 0.46$, then we need to find $\sin C = \sin(180° - D)$.

$$\sin \theta = \sin 180° - \theta$$

$$\sin C = \sin 180° - D = \sin D = 0.46$$

3. Mickey wants to make a right kite (a kite with two opposite right angles). He has a piece of cloth for which he managed to measure the short sides, 26 cm, and the cosine of the smallest angle, 0.8. Find the width of the frame, which should join the right angles of the kite.

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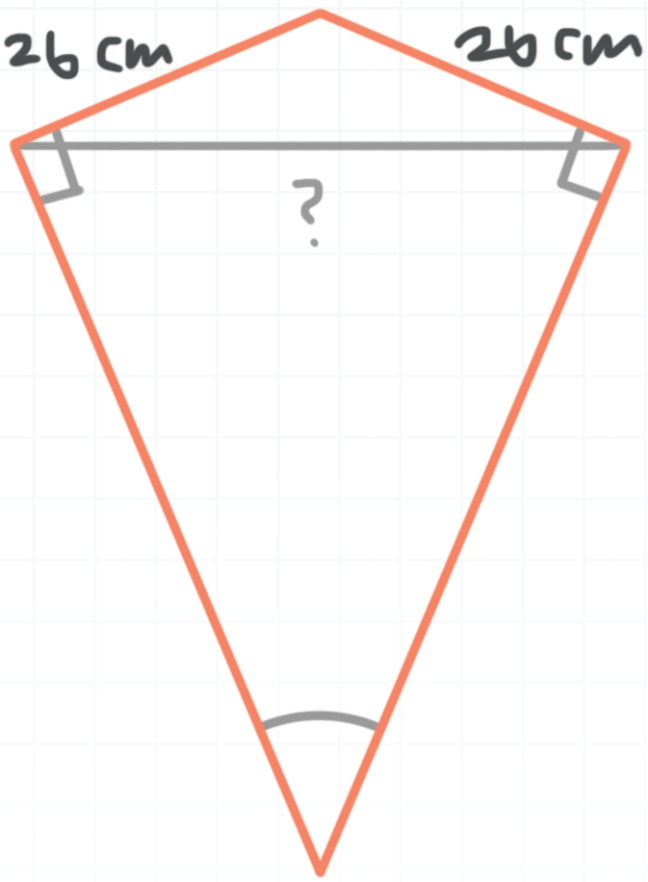
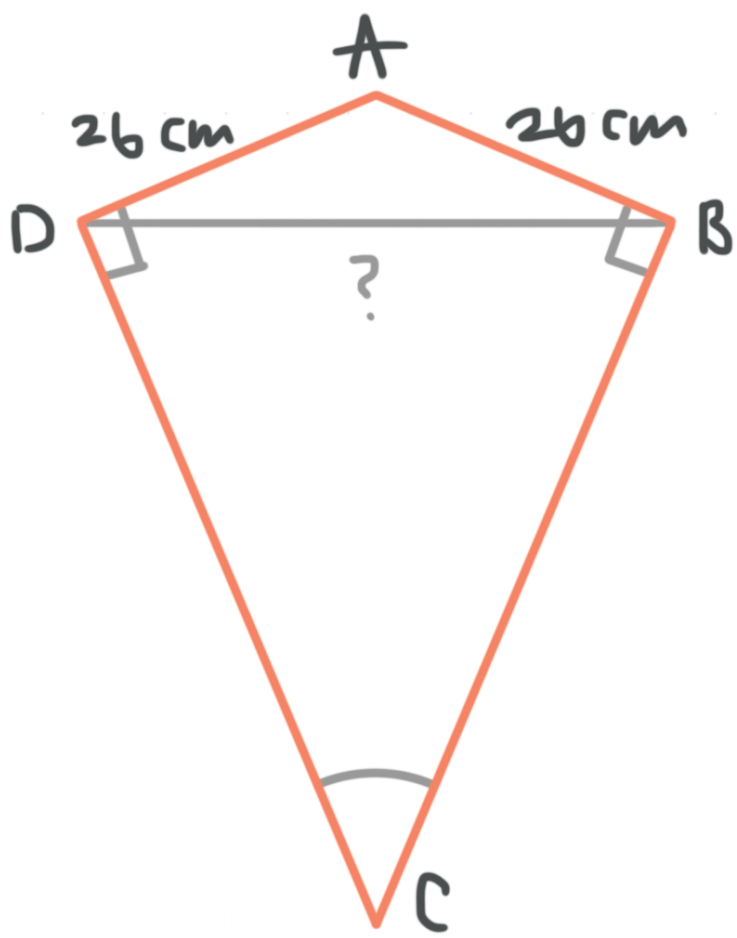


Solution:

Call the kite $ABCD$, where the angles $B = D = 90°$, the short sides are $AB = AD = 26$ cm, and $\cos C = 0.8$. So BD is the length we need to find.



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Our goal is to find BD from the triangle ABD using the law of cosines. We know two legs of the triangle, so we just need the cosine of the angle between them, $m\angle A$.

Since in a quadrilateral the sum of all angles is $360°$,

$$A + B + C + D = 360°$$

The angles B and D are right, so

$$A + 90° + C + 90° = 360°$$

$$A + C = 180°$$

$$A = 180° - C$$

So knowing $\cos C = 0.8$, we need to find $\cos(180° - C)$. The angle C lies in quadrant I, so its reference angle is C. The angle $180° - C$ lies in quadrant II, so its reference angle is

$$180° - (180° - C) = C$$

Since the angles C and $180° - C$ have equal reference angles, by the reference angle theorem they have the same absolute values of their trig functions (other than the fact that they might have different signs).

Since $180° - C$ lies in quadrant II, its cosine is negative, and its absolute value is equal to $\cos C = 0.8$, so

$$\cos A = \cos(180° - C) = -\cos C = -0.8$$

Now we can apply the law of cosines for triangle ABD.

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$$BD^2 = AB^2 + AD^2 - 2 \cdot AB \cdot AD \cdot \cos A$$

$$BD = \sqrt{AB^2 + AD^2 - 2 \cdot AB \cdot AD \cdot \cos A}$$

$$BD = \sqrt{26^2 + 26^2 - 2 \cdot 26 \cdot 26 \cdot (-0.8)} \approx 49 \text{ cm}$$

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GIVEN A POINT ON THE TERMINAL SIDE OF THE ANGLE

■ 1. On the circle with center at the origin and radius 5, find the point(s) where $\cos\theta = -0.6$.

Solution:

We'll use the formula for cosine.

$$\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\frac{x}{\sqrt{x^2 + y^2}} = -0.6$$

Because the circle has radius 5,

$$\frac{x}{5} = -0.6$$

$$x = -3$$

So the points we're looking for are the points where the vertical line $x = -3$ intersects the circle centered at the origin with radius 5.

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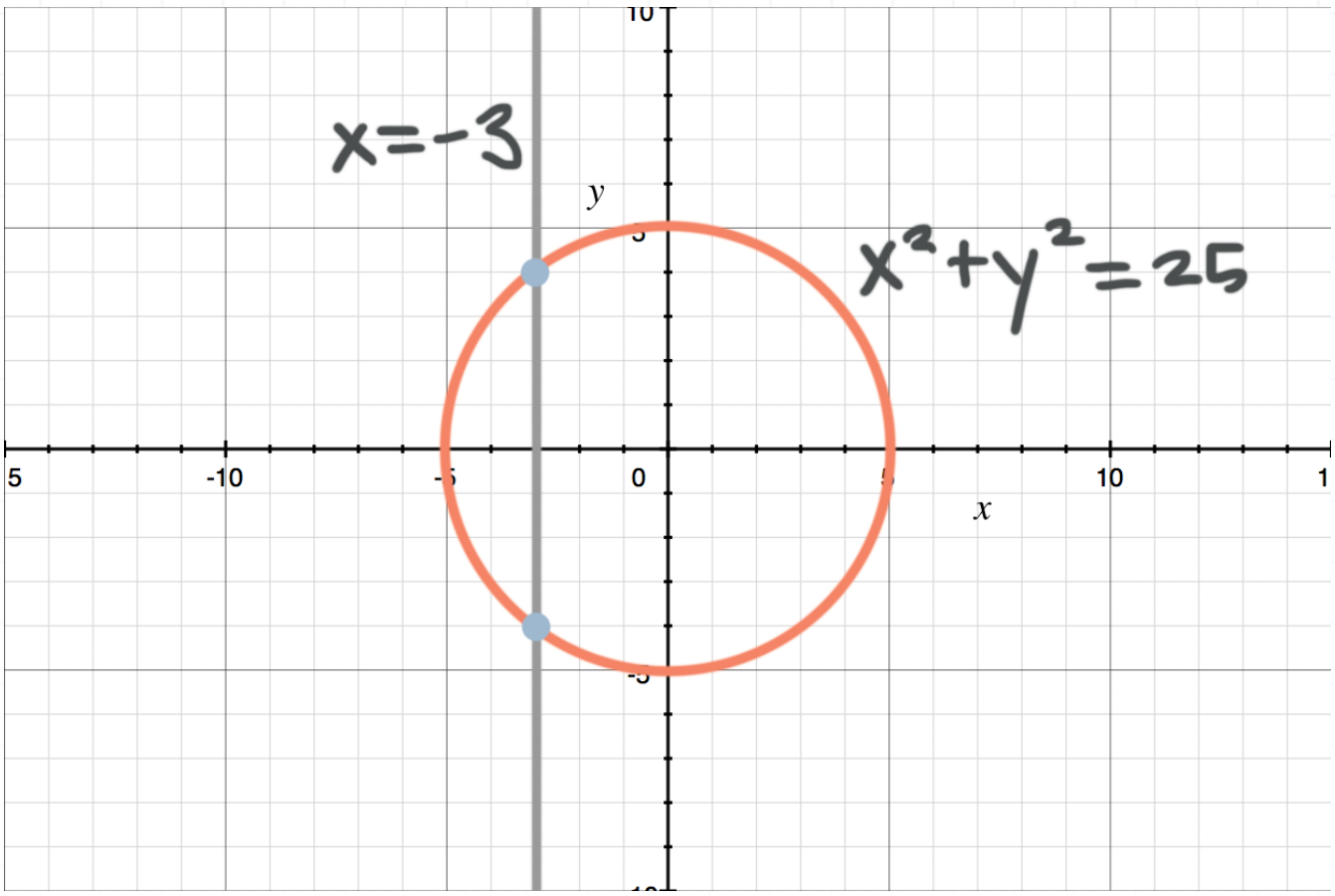

Substitute $x = -3$ into the equation of the circle.

$$x^2 + y^2 = 25$$

$$(-3)^2 + y^2 = 25$$

$$y^2 = 16$$

$$y = \pm 4$$

So the points of intersection are $(-3, -4)$ and $(-3,4)$.

■ 2. Find the point P on the line $x + y = 1$ where sine of the angle between P and the positive direction of the x-axis is $\sqrt{2}/2$.

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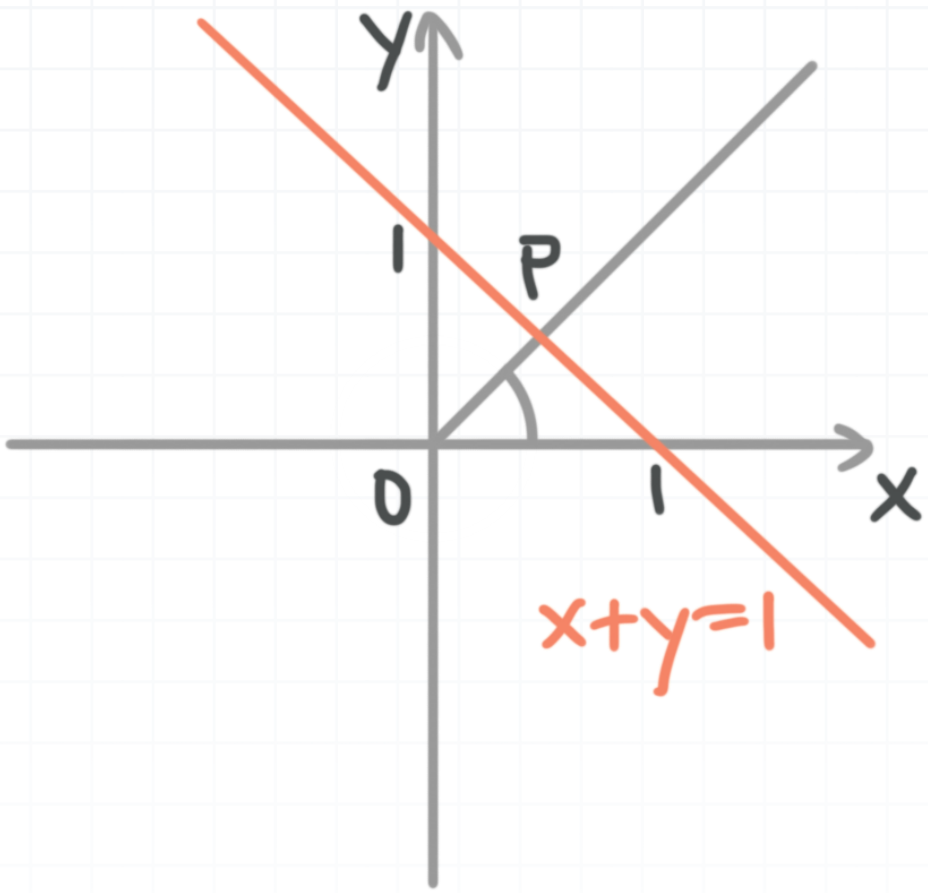

Solution:

We know that the sine of an angle is given by

$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}}$$

$$\frac{y}{\sqrt{x^2+y^2}} = \frac{\sqrt{2}}{2}$$

Since P lies on the line, its coordinates satisfy the equation of the line $x+y=1$, so $x = 1-y$. Substitute $x = 1-y$ into the equation.

$$\frac{y}{\sqrt{(1-y)^2+y^2}} = \frac{\sqrt{2}}{2}$$

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$$\frac{y^2}{(1-y)^2+y^2}=\frac{1}{2}$$

$$2y^2=(1-y)^2+y^2$$

$$2y^2=1-2y+y^2+y^2$$

$$1-2y=0$$

$$y=0.5$$

Substitute $y=0.5$ into $x=1-y$.

$$x=1-0.5$$

$$x=0.5$$

3. An airplane flew 3 hours North-North-East (NNE) with a constant velocity of 120 km/h. How far north is the plane from the airport and how far east is the plane from the airport?

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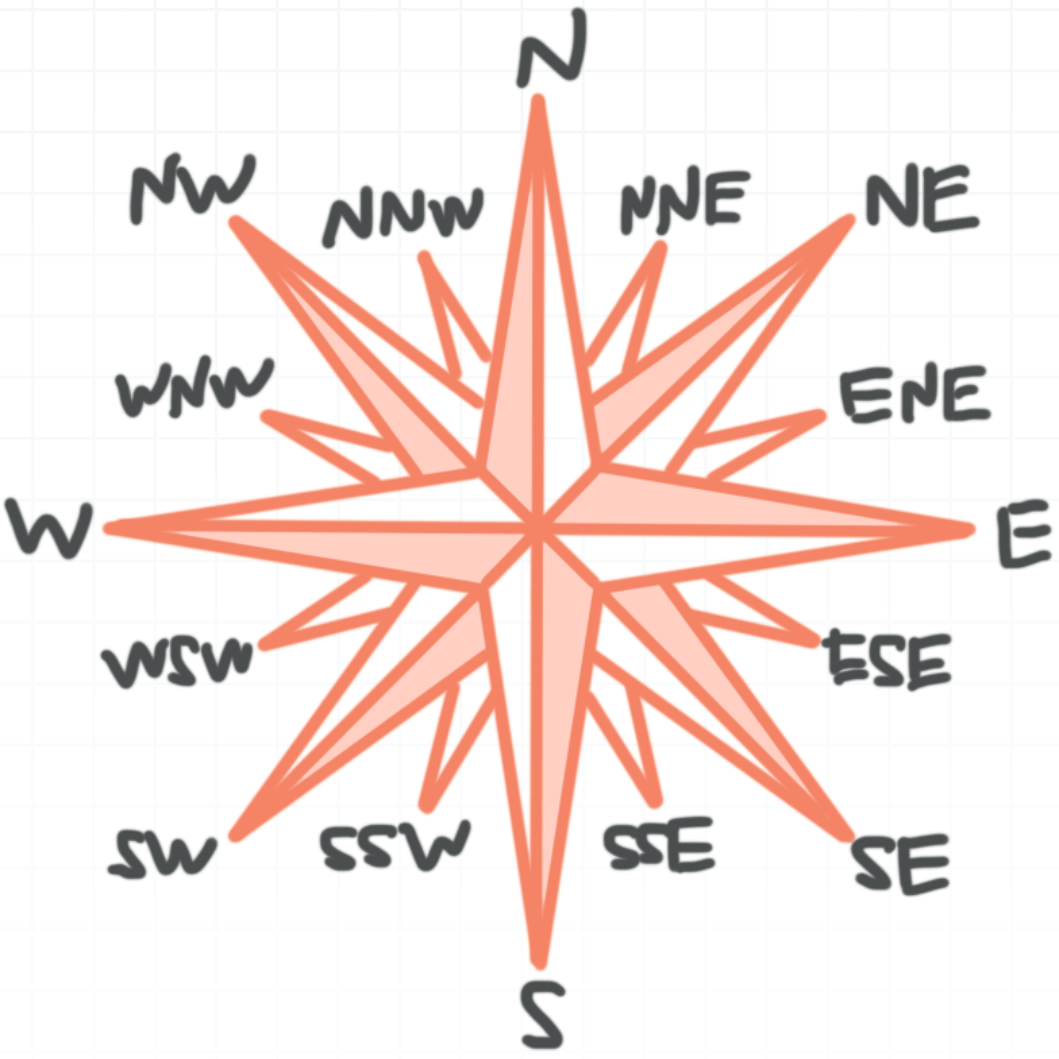

Solution:

A direction of NNE is the angle

$$\frac{3}{4} \cdot 90^\circ = 67.5^\circ$$

The airplane flew $3 \cdot 120 = 360$ km from the airport. So let's model the problem using the coordinate system where the airport is at the origin, the x-axis is directly east, the y-axis is directly north, and one unit is 1 km.

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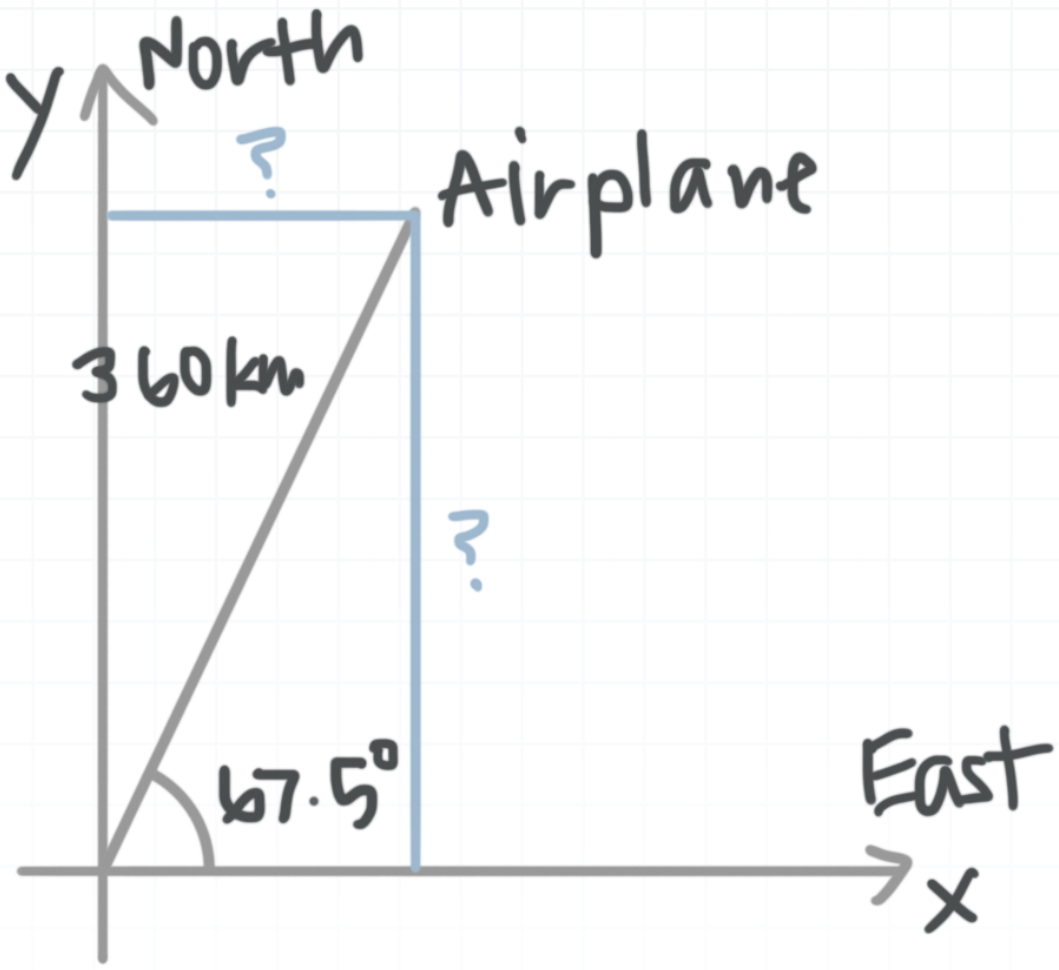

We need to find the coordinates (x, y) of the airplane, given the angle $\theta = 67.5°$ and distance from the origin $D = 360$ km. We'll use the formulas for sine and cosine.

$$\sin\theta = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\sin 67.5° = \frac{y}{360}$$

$$y = 360 \sin 67.5° \approx 333 \text{ km}$$

and

$$\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$$

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$$\cos 67.5^\circ = \frac{x}{360}$$

$$x = 360 \cos 67.5^\circ \approx 138 \text{ km}$$

So the airplane is 333 km north of the airport and 138 km east of the airport.

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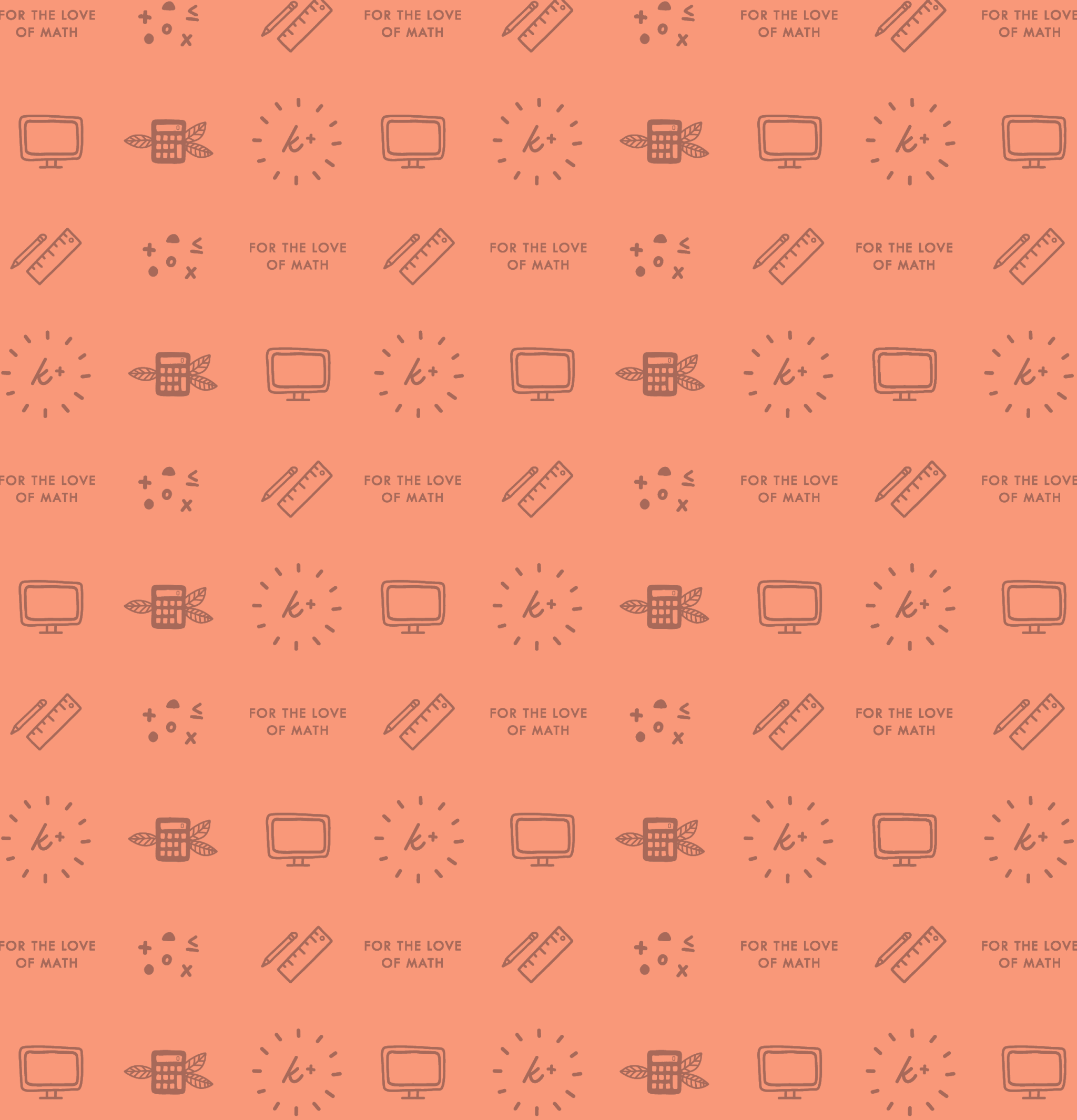

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